Power link budget for propagating Bessel beams

Mauro Ettorre, Massimiliano Casaletti, Anthony Grbic

Power Link Budget for Propagating Bessel Beams

Mauro Ettorre, Massimiliano Casaletti
Institut d'Electronique et de Télécommunications de Rennes
UMR CNRS 6164, Université de Rennes 1
35042 Rennes Cedex, France
mauro.ettorre, massimiliano.casaletti@univ-rennes1.fr

Anthony Grbic
Department of Electrical Engineering and Computer Science
Radiation Laboratory, University of Michigan
Ann Arbor, MI 48109-2122, USA
agrbic@umich.edu

***Abstract*—The power link budget for a system that transmits and receives propagating Bessel beams is studied. The transmitter and receiver are separated by a distance D and consist of leaky radial waveguides. Full-wave simulations are used to compute the admittance-matrix representation of the system. The resonances of the coupled transmitter and receiver are then derived using classical network theory. For comparison purposes, a second configuration with its transmitter and receiver connected by a circular waveguide is considered. In contrast to the open system, such a configuration is closed and does not radiate. It is found that within the non-diffractive range of the Bessel beam, both closed and open systems exhibit the same resonances within an error of 0.6%. Calculations show that the power efficiency of the open system can exceed 85% within the non-diffractive range. The proposed system may find application in areas such as wireless power transfer, near-field communication and non-destructive evaluation.**

I. Introduction

In optics, propagating Bessel beams are defined as solutions to the scalar wave equation that remain confined and do not undergo diffractive spreading [1]. They are of the form: $E(\rho, z) = J_0(k_\rho \rho)e^{-jk_z z}$, where $k_\rho^2 + k_z^2 = k_0^2$. The free-space propagation constant is denoted by k_0, (ρ, z) are coordinates of the cylindrical system, and J_0 is the zeroth-order Bessel function of the first kind. The spatial spectrum of Bessel beams consists of a single ring in the spectral domain. Therefore, propagating Bessel beams are characterized by their transverse wavenumber: $0 \leq k_\rho \leq k_0$.

Recently, two papers by the authors [2], [3] have shown that it is possible to generate propagating Bessel beams in the microwave frequency range using planar structures. In particular, propagating Bessel beams are generated using a leaky radial waveguide. The waveguide consists of an impedance sheet above a ground plane, fed by a coaxial probe (see Fig. 1). The resulting structure is simple to fabricate, planar and extremely compact in contrast to earlier structures [4]-[6].

In the present work, a system consisting of two identical leaky radial waveguides (transmitter and receiver) is considered. The link budget associated with this system, capable of generating and receiving propagating Bessel beams, is found. The transmitter and receiver are axially aligned and separated by a distance D (see Fig. 1) in free space. Full-wave simulations are used to characterize the system in terms of two-port network parameters. This is in contrast to earlier works on Bessel beams which were based on approximate order-of-magnitude calculations [7]. For comparison purposes, a second

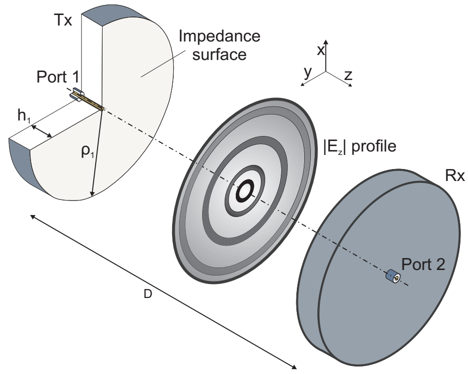

Fig. 1. Schematic of the propagating Bessel beam system. The transmitter and receiver are leaky radial waveguides. The leaky radial waveguide, with a metallic rim at ρ_1, consists of an impedance surface above a ground plane. The transmitter and receiver are axially aligned and separated by a distance D in free space. The Bessel profile for the E_z component is graphically shown in the figure.

configuration of identical, leaky radial waveguides is also considered. It consists of two leaky radial waveguides directly connected by a circular waveguide (with radial dimension ρ_1). The same resonances are found for the two configurations within the non-diffractive range of the generated Bessel beam. The power efficiency of the original system is evaluated using an accurate network approach. Efficiencies of greater than 85% at 10 GHz are possible within the non-diffractive range with a simultaneous conjugate match at receiver and transmitter, and proper Bessel beam tuning. Such high efficiencies are attributed to the focusing behavior and reduced radiation losses of propagating Bessel beams.

II. Leaky-Wave Radial Waveguide

The design procedure for the leaky-wave radial waveguides (transmitter and receiver) is provided in [2], [3]. As shown in these references, the zeroth-order Bessel profile is imposed on the z component of the electric field. Due to the symmetry of the structure only transverse magnetic-polarized (TM) Bessel beams (with respect to z in Fig. 1) are considered. The design parameters for the leaky radial waveguide are the frequency of operation f_0, the transverse complex wavenumber $k_\rho = \beta_\rho - j\alpha_\rho$ of the desired Bessel beam and the non-diffractive range. The value of the impedance sheet (Z_s), height (h_1) and radius (ρ_1) of the circular waveguide are then derived (see

Fig. 1). In the present case, we assume: $f_0 = 10$ GHz and $k_\rho = (0.83 - j0.022)k_0$ and the non-diffractive range is $z < 2\lambda_0$. Therefore, it follows that $Z_s = -25j\ \Omega$, $h_1 = 1$ mm and $\rho_1 = 2.96\lambda_0 = 88.89$ mm (λ_0 and k_0 are the free space wavelength and wavenumber at $f = 10$ GHz, respectively). The behavior of the radial waveguide as a Bessel beam launcher was verified through full-wave simulations [8]. The E_z component of the electric field was derived within the non-diffractive range of the leaky radial waveguide, as shown in Fig. 2. For small distances ($z < 1.25\lambda_0$), the shape of the Bessel beam profile remains nearly unchanged with distance, as expected for a Bessel beam. Further away from the aperture, the Bessel beam shape is preserved only in the main beam and increased side lobes appear.

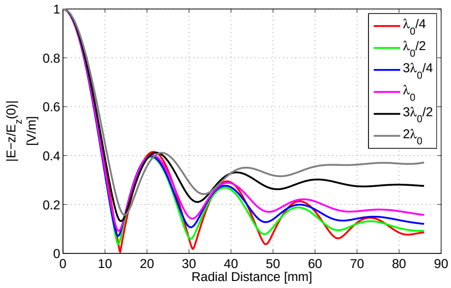

Fig. 2. Normalized E_z component at different distances above the leaky radial waveguide: $z = 0.5$-$2\lambda_0$.

III. Free Resonance and Power Efficiency of the System

The link budget for the system shown in Fig. 1 is evaluated. The ports of the system correspond to the coaxial feeds. Full-wave simulations are used to derive the admittance matrix (Y-matrix) of the system for different separation distances D [8] of transmitter and receiver. It should be noted that the Y-matrix representation takes into account the radiation losses associated with the system. A closed structure consisting of two leaky radial waveguides directly connected by a circular waveguide is also considered for comparison purposes. The circular waveguide is chosen to have the same radial dimension, ρ_1, as the leaky radial waveguides (see Fig. 1). This ensures that the circular waveguide wall coincides with a zero of the E_z Bessel profile generated by the leaky-radial waveguides. Therefore, the circular waveguide should not perturb the propagation of the Bessel beams. It is worth noting that the closed configuration eliminates radiation loss and therefore represents the most efficient power transfer possible between the two Bessel beam launchers. The equivalence between the two configurations is verified by deriving their free resonances. In particular the zeros of the determinant of the Y-matrix are derived for the open structure, whereas a transverse resonance technique is adopted for the closed case. The same resonances are found for the two configurations within an error of 0.6% for distances within the non-diffractive range of the generated Bessel beam. This confirms that that both configurations are equivalent thanks to the accurate generation of Bessel beams by the leaky radial waveguide. To further validate this observation, the power efficiency of the open configuration is computed from the Y-matrix using standard network theory. The power efficiency is defined as the ratio of power delivered to the load to the power available from the source [9]. A simultaneous conjugate match is imposed on the two ports for maximum power transfer [9], [10]. The efficiency can exceed 85% at the operating frequency of 10 GHz, within the non-diffractive range of the Bessel beam. However, proper tuning of the Bessel launcher is needed to take into account the frequency shift associated with the coupling between transmitter and receiver. This high efficiency is due to the focusing behavior and reduced radiation losses of propagating Bessel beams.

IV. Conclusion

The power link budget for propagating Bessel beams has been studied. The propagating Bessel beams are generated using leaky radial waveguides. Full-wave simulations and classical network theory are used to evaluate the power efficiency and resonances of the system. It is found that the system is equivalent to a closed system consisting of a circular waveguide connecting two leaky radial waveguides. In addition, an efficiency in excess of 85% is reported within the non-diffractive range of the generated Bessel beam. The proposed system consisting of a Bessel beam transmitter and reciever may find application in areas such as wireless power transfer, near-field communication and non-destructive evaluation.

Acknowledgement

M. Ettorre would like to thank the University of Rennes 1, Rennes, France, for its support and financial contribution. A. Grbic would like to acknowledge the support of the Presidential Early Career Award for Scientists and Engineers (FA9550-09-1-0696).

References

[1] J. Durnin, “Exact Solutions for nondiffracting Beams. I. The scalar theory,” *J. Opt. Soc. Am. A.*, vol. 4, no. 4, pp. 651-654, Apr. 1987.

[2] M. Ettorre, and A. Grbic, “Generation of Propagating Bessel Beams using Leaky-Wave Modes,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3605-3613, Aug. 2012.

[3] M. Ettorre, S. M. Rudolph, and A. Grbic, “Generation of Propagating Bessel Beams using Leaky-Wave Modes: experimental validation,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 2645-2653, June 2012.

[4] S. Monk, J. Arlt, D. A. Robertson, J. Courtial, and M. J. Padgett, “The Generation of Bessel beams at millimetre-wave frequencies by use of an axicon,” *Opt. Commun.*, vol. 170, pp. 213-215, Nov. 2000.

[5] Z. Li, K. B. Alici, H. Caglayan, and E. Ozbay, “Generation of an Axially Asymmetric Bessel-Like Beam from a Metallic Subwavelength Aperture,” *Phys. Rev. Lett.*, vol. 102, no. 14, Apr. 2009.

[6] M. A. Salem, A. H. Kamel, and E. Niver, “Microwave Bessel beams generation using guided modes,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 2241-2247, June 2011.

[7] J. Durnin, J. J. Miceli, Jr., and J. H. Eberly, “Comparison of Bessel and Gaussian beams,” *Opt. Lett.*, vol. 13, no. 2, pp. 79-80, Feb. 1988.

[8] Comsol Multiphysics version 4.2, 1997-2012 COMSOL AB.

[9] D. M. Pozar, *Microwave Engineering*, 2nd ed., John Wiley & Sons, Inc., New York, 1998.

[10] M. Ettorre, and A. Grbic, “A transponder-based, nonradiative wireless power transfer,” *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 1150-1153, Dec. 2012.